Intermediate Algebra | 2e

Connecting Concepts Through Applications

Mark Clark
Cynthia Anfinson

Second Edition

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Mark Clark
PALOMAR COLLEGE

Cynthia Anfinson
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CENGAGE

Australia • Brazil • Mexico • Singapore • United Kingdom • United States

To my wife Christine for her love and support throughout our lives together. And to our children Will and Rosemary.

MC

To my husband Fred and son Sean, thank you for your love and support.

CA

Extra thanks go to Jim and Mary Eninger for opening up their family cabin for me to spend many a day and night writing. Also to Dedad and Mimi, who built the cabin in the 1940s.

MC

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Second Edition
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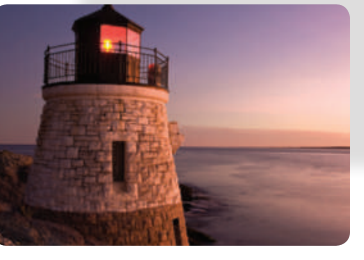

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Preface

In teaching the Intermediate Algebra course, we wanted to help our students apply traditional mathematical skills in real-world contexts. This text is our attempt to address the challenge of supplying applications that students relate to and lay a strong math foundation. This text is the result of many years of using this approach in our classrooms, refined with the help of many of you who shared your thoughtful feedback with us throughout our extensive revision process.

Our *application-driven* approach is designed to engage students while they master algebraic concepts, critical thinking, and communication skills. Our goal is simple: to present mathematics as concepts in action rather than as a series of techniques to memorize and to have students *understand* and *connect* with mathematics—*what it means in real-world contexts*—while developing a solid foundation in algebra.

About The Authors

MARK CLARK

Mark Clark graduated from California State University, Long Beach, with a Bachelor's and Master's in Mathematics. He is a full-time Associate Professor at Palomar College and has taught there since 1996. He is committed to teaching his students through applications and using technology to help them both understand the mathematics in context and communicate their results clearly. Intermediate algebra is one of his favorite courses to teach, and he continues to teach several sections of this course each year. Mark also loves to share his passion for teaching concepts of developmental math through applications by giving workshops and talks to other instructors at local and national conferences.

CINDY ANFINSON

Cindy Anfinson graduated from the University of California, San Diego, with a Bachelor of Arts Degree in Mathematics and is a member of Phi Beta Kappa. Under the Army Science and Technology Graduate Fellowship, she earned a Master of Science Degree in Applied Mathematics from Cornell University. She is currently an Associate Professor of Mathematics at Palomar College. At Palomar College, she has worked with the First-Year Experience and the Summer Bridge programs, she was the Mathematics Learning Center Director for a 3-year term, and has served on multiple committees, including the Basic Skills Committee and the Student Success and Equity Council.

New to This Edition

Four Toolboxes are included throughout the new edition: the Equations Solving Toolbox, the Factoring Toolbox, the Expression Simplifying Toolbox, and the Modeling Toolbox. These Toolboxes are integrated throughout the text with visual icons so that students receive just-in-time help connecting them to the solving techniques and tools used for different types of problems. Each Toolbox emphasizes how these fundamental tools are used throughout the course. A quick reference of all four Toolboxes as well as the Geometric Formulas and Unit Conversions appears at the back of the text. The text highlights the importance of using these from the start of the course.

There is an increased emphasis on students identifying equation and function types within solving, graphing, and modeling problems. This helps students to review previous material and connect it to the current topics.

Students are asked to provide reasons for each step they took in selected Solving exercises. This helps students to think critically about and explain the Solving process, and it connects with the Toolboxes that have been integrated throughout the text.

Vocabulary short-answer exercises have been added at the start of each exercise set. These help students learn the vocabulary of algebra and improve their communication skills.

The Exercise Sets have been updated with new data and current applications to help students see connections between mathematics and the world in which they live.

Within WebAssign, there is expanded problem coverage with an emphasis on conceptual problems, full "WatchIt" coverage with closed-captioning, and expanded "MasterIt" and "Expanded Problem" coverage with emphasis on conceptual problems.

The Annotated Instructor's Edition has been replaced with a comprehensive Instructor's Manual. Practical tips and classroom examples are provided on how to approach and pace chapters as well as integrate features such as Concept Investigations into the classroom. For every student example in the student text, there is a different instructor classroom example with accompanying answers that can be used for additional in-class practice and/or homework.

Activities that are hand-selected by the authors provide additional opportunities for Instructors to get their students involved using active learning in the classroom. WebAssign suggestions integrated throughout the Instructor Manual also tie in the digital aspects of the course so that no matter how instructors approach their class, they feel supported.

Ancillaries

For the Student

Online Student Solutions Manual

(ISBN: 978-1-337-61561-7)

Author: Scott Barnett

The Student Solutions Manual provides worked-out solutions to all of the odd-numbered exercises in the text.

STUDENT

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Online Complete Solutions Manual

(ISBN: 978-1-337-61560-0)

Author: Scott Barnett

The Complete Solutions Manual provides worked-out solutions to all of the problems in the text.

Instructor's Companion Website

Everything you need for your course in one place! Access and download a comprehensive Instructor's Manual that paces the chapters, includes WebAssign suggestions, and provides additional opportunities for in-class practice, homework, and activities. In addition, you can find the online chapter, PowerPoint presentations, and more on the companion site. This collection of book-specific lecture and class tools is available online via www.cengage.com/login.

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Mark Clark
Cindy Anfinson

Linear Functions

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The U.S. Census Bureau found that the average number of square feet of floor area in new one-family houses in 1980 was 1740 square feet. In 2015, the average number of square feet of floor area increased to 2687 square feet. In this chapter, we will discuss how to use linear models to analyze trends in real-life data. One of the chapter projects will ask you to investigate the costs associated with installing new flooring in a home.

1.1 Solving Linear Equations

LEARNING OBJECTIVES

- Solve linear equations.
- Write complete answers to application problems.
- Determine whether a solution is reasonable.
- Solve literal equations.

Variables are used to represent quantities that can change or are unknown. In application problems, it is important to define variables so the reader knows what the variables represent. In traditional algebra problems, it is common to use the variables x and y. In application problems, variables often are given a letter that closely matches what the variable represents. For example, if a problem is discussing the weekly pay of a student tutor, then the variable P can represent the weekly pay in dollars.

Variables are also identified as *input* or *output* variables. The input variable can be thought of as the information that is being entered into the problem. The output variable is the information that results, or is produced by, the problem. The input variable is also called the *independent* variable. The output variable is also called the *dependent* variable. In traditional algebra problems, x is typically the input (or independent) variable. The variable y is the output (or dependent) variable. In application problems, we need to think about the effect of the variable to determine whether it is the input or output variable. For example, suppose a student tutor is paid $10.00 per hour. An equation that computes the tutor's weekly pay is

$$P = 10h$$

where $h =$ the number of hours worked per week and $P =$ the weekly pay in dollars. Here h is the input variable, as it is the information that must be entered into the problem in order to compute the weekly pay. P is the output variable, as it is the information that is produced by the problem (that is, it is the output).

Equations can be used to represent many real-life situations. One of the uses of algebra is to solve equations for an unknown quantity, or variable. In this section, you will learn how to solve linear equations for a missing variable and how to write a complete solution. A complete solution includes a sentence that gives the units (how a quantity is measured, such as in dollars) and the meaning of the solution in that situation. Providing these details demonstrates a clear understanding of the solution.

Example 1 Solving applications and providing complete solutions

U-Haul charges $19.95 for the day and $0.79 per mile driven to rent a 10-foot truck. The total cost to rent a 10-foot truck for the day can be represented by the equation

$$U = 19.95 + 0.79m$$

where U is the total cost in dollars to rent a 10-foot truck from U-Haul for the day and m is the number of miles the truck is driven.

a. State the given variables and their definitions.

b. Determine how much it will cost to rent a 10-foot truck from U-Haul and drive it 75 miles.

c. Determine the number of miles you can travel for a total cost of $175.00.

SOLUTION

a. $m =$ the number of miles driven
$U =$ the total cost in dollars to rent a 10-foot truck from U-Haul for the day

b. Because the number of miles driven was given, replace the variable m in the equation with the number 75 and solve for the missing variable U as follows:

$$U = 19.95 + 0.79m$$
$$U = 19.95 + 0.79(75)$$
$$U = 19.95 + 59.25$$
$$U = 79.20$$

This answer indicates that renting a 10-foot truck from U-Haul for the day and driving it 75 miles will cost $79.20.

c. Because the total cost of $175.00 is given in the statement, substitute 175.00 for the variable U and solve for the missing variable m.

$$U = 19.95 + 0.79m$$
$$175.00 = 19.95 + 0.79m$$
$$\underline{-19.95 \quad -19.95} \qquad \text{Subtract 19.95 from both sides.}$$
$$155.05 = 0.79m$$
$$\frac{155.05}{0.79} = \frac{0.79m}{0.79} \qquad \text{Divide both sides by 0.79.}$$
$$196.266 \approx m$$

Since U-Haul would charge for a full mile for the 0.266, round down to 196 miles to stay within the budget of $175. Check this answer by substituting $m = 196$ to be sure U will equal $175.

$$U = 19.95 + 0.79(196)$$
$$U = 19.95 + 154.84$$
$$U = 174.79$$

This answer indicates that for a cost of $175.00, you can rent a 10-foot truck from U-Haul for a day and drive it 196 miles.

PRACTICE PROBLEM FOR EXAMPLE 1

While you are on spring break in Fort Lauderdale, Florida, the cost, C, in dollars of your taxi ride from the airport to your hotel can be represented by the equation

$$C = 2.10 + 2.40m$$

when the ride is m miles long.

a. Define the variables.

b. What will an 8-mile taxi ride cost?

c. How many miles can you ride if you budget $35 for the taxi?

(*Note:* The answers to the Practice Problems are in Appendix D.)

Many problems in this book will investigate applications that involve money and business. Defining some business terms will help in understanding the problems and explaining the solutions. Three main concepts in business are **revenue**, **cost**, and **profit**.

Revenue is the total amount of money that is brought into a business through sales. For example, if a pizza place sells 10 pizzas for $12 each, the revenue would

Connecting the Concepts

What is the difference between the equal (=) symbol and the approximation (≈) symbol?

In mathematics, we use these symbols and others to show a relationship between two quantities or between two expressions.

The equal sign (=) is used when two quantities or expressions are equal and exactly the same.

The approximation symbol (≈) is used to show that two quantities or expressions are approximately the same. The approximation symbol will be used whenever a quantity is rounded.

Connecting the Concepts

How are we going to round?

In general, we will round values to at least one more decimal place than the given numbers in the problem.

In some applications, rounding will be determined by what makes sense in the situation.

When a specific rounding rule is given in a problem, we will follow that rule.

be $10 \text{ pizzas} \cdot \$12 \text{ per pizza} = \$120$. Revenue is often calculated as price times the quantity sold. The revenue for a business cannot be a negative number.

Cost is defined as the amount of money paid out for expenses. Expenses often are categorized in two ways: fixed costs and variable costs. The same pizza place might have fixed costs such as rent, utilities, and salaries. It would have variable costs of supplies and food ingredients depending on the number of pizzas made. The cost for the business would be the fixed costs and the variable costs added together.

The profit for a business is the revenue minus the cost. If this pizza place had a cost of \$100 when making the 10 pizzas, they would have a profit of $\$120 - \$100 = \$20$. Although a business cannot have a negative revenue, profit can be negative. When profit is negative, it is sometimes called a loss.

The **break-even** point of a business is the point at which the revenue from a product is the same as the cost. The break-even point also occurs when the profit is zero. For a company that is considering a new product and wants to know how many should be produced or sold to start making a profit, the point at which profit changes from negative to positive is important to know.

DEFINITIONS

Revenue The amount of money brought into a business through sales. Revenue is often calculated as

$$\text{revenue} = \text{price} \cdot \text{quantity sold}$$

Cost The amount of money spent by a business to create and/or sell a product. Cost usually includes both fixed costs and variable costs. Fixed costs are the same each month or year, and variable costs change depending on the number of items produced and/or sold.

$$\text{cost} = \text{fixed cost} + \text{variable cost}$$

or

$$\text{cost} = \text{fixed cost} + \text{cost per item} \cdot \text{quantity sold}$$

Profit The amount of money left after all costs.

$$\text{profit} = \text{revenue} - \text{cost}$$

Break-even point A company breaks even when their revenue equals their cost or when their profit is zero.

$$\text{revenue} = \text{cost}$$
$$\text{profit} = 0$$

What's That Mean?

Mathematical Verbs

Simplify: Use arithmetic and basic algebra rules to make an expression simpler. ***We simplify expressions.***

Evaluate: Substitute any given values for variables and simplify the resulting expression or equation.

Solve: Isolate the given variable in an equation on one side of the equal sign using the properties of equality. This will result in a value that the isolated variable is equal to. ***We solve equations.***

Example 2 Revenue, cost, and profit

A small bicycle company produces high-tech bikes for international race teams. The company has fixed costs of \$54,500 per month for rent, salaries, and utilities. For every bike they produce, it costs them \$5750 in materials and other expenses related to that bike. The company can sell each bike for an average price of \$13,995, but it can produce a maximum of only 20 bikes per month.

a. Write an equation for the monthly cost of producing b bikes.

b. How much does it cost the bicycle company to produce 20 bikes in a month?

c. Write an equation for the monthly revenue from selling b bikes.

d. How much revenue will the bicycle company make if they sell 10 bikes in a month?

e. Write an equation for the monthly profit the company makes if they produce and sell b bikes. (You can assume that they will sell all the bikes they make.)

f. What is the profit of producing and selling 15 bikes in a month?

g. How many bikes does the company have to produce and sell in a month to make \$45,000 profit?

h. How many bikes does the company have to produce and sell in a month to make \$150,000 profit?

SOLUTION

a. Define the variables for the problem.

b = The number of bikes produced each month. (Remember that a maximum of 20 bikes can be produced each month.)

C = The monthly cost, in dollars, to produce b bikes.

Each bike cost \$5750 for materials and other expenses, so multiply b by 5750, and then add on the fixed costs, to get the total monthly cost. This gives the following equation.

$$C = 5750b + 54500$$

b. The number of bikes produced is given. Substitute 20 for b and simplify the right side to find C.

$$C = 5750b + 54500$$
$$C = 5750(20) + 54500$$
$$C = 169500$$

A monthly production of 20 bikes will result in a total monthly cost of \$169,500.

c. Define the variables for the problem. Recall that b was already defined in part a.

b = The number of bikes produced each month.

R = The monthly revenue, in dollars, from selling b bikes.

The bicycle company can sell each bike for an average price of \$13,995, so the revenue can be calculated by using the equation

$$R = 13995b$$

d. The number of bikes is given, so substitute 10 for b.

$$R = 13995b$$
$$R = 13995(10)$$
$$R = 139950$$

The total monthly revenue from selling 10 bikes is \$139,950.

e. Profit is calculated by taking the revenue and subtracting any business costs.

b = The number of bikes produced each month.

P = The monthly profit, in dollars, from producing and selling b bikes.

Use the equations for revenue and cost written earlier.

$$P = R - C$$
$$P = (13995b) - (5750b + 54500) \quad \text{Substitute for } R \text{ and } C$$

Skill Connection

Using the Distributive Property in Subtraction

When subtracting an expression in parentheses, remember that subtraction can be represented as adding the opposite.

$$a - b = a + (-1)b$$

Since subtraction is defined as adding the opposite, we see that we use the distributive property first to distribute the factor of -1. This will ensure that we subtract all the terms of the second expression.

$$\begin{aligned} &25x - (5x + 30) \\ &= 25x + (-1)(5x + 30) \\ &= 25x - 5x - 30 \\ &= 20x - 30 \end{aligned}$$

Often, we do this without writing the -1. We will say to distribute the negative sign, implying the -1 that is not seen in the original expression or equation.

Simplify by distributing the sign and combining like terms.

$$P = (13995b) - (5750b + 54500)$$
$$P = 13995b - 5750b - 54500 \quad \text{Distribute the negative sign.}$$
$$P = 8245b - 54500 \quad \text{Combine like terms.}$$

f. The number of bikes is given, so substitute 15 for b.

$$P = 8245(15) - 54500$$
$$P = 69175$$

The monthly profit from producing and selling 15 bikes is \$69,175.

g. The amount of profit desired is given. Substitute 45,000 for P and solve for b.

$$\begin{aligned} P &= 8245b - 54500 \\ 45000 &= 8245b - 54500 \\ +54500 & \qquad\quad +54500 \quad \text{Add 54,500 to both sides.} \\ 99500 &= 8245b \\ \frac{99500}{8245} &= \frac{8245b}{8245} \quad \text{Divide both sides by 8245.} \\ 12.068 &\approx b \end{aligned}$$

As there cannot be 12.068 bikes, let's compare profits for 12 bikes and 13 bikes.

$$P = 8245(12) - 54500 \qquad P = 8245(13) - 54500$$
$$P = 44440 \qquad P = 52685$$

To make at least \$45,000 profit for the month, the company will need to produce at least 13 bikes.

Since producing 12 bikes does not quite make \$45,000 profit, round up to the next whole number quantity of bikes.

h. The amount of profit desired is given. Substitute 150,000 for P and solve for b.

$$\begin{aligned} P &= 8245b - 54500 \\ 150000 &= 8245b - 54500 \\ +54500 & \qquad\quad +54500 \quad \text{Add 54500 to both sides.} \\ 204500 &= 8245b \\ \frac{204500}{8245} &= \frac{8245b}{8245} \quad \text{Divide both sides by 8245.} \\ 24.803 &\approx b \end{aligned}$$

Check this answer by substituting in 24.803 for b and confirming that it gives a profit of \$150,000.

$$P = 8245(24.803) - 54500$$
$$P = 204500.735 - 54500$$
$$P = 150000.735$$

The answer was rounded, so the check is not exact, but it is close enough.

Again, there is a decimal answer, so round to the whole number of bikes that will produce the desired profit. That would give 25 bikes produced in a month. However, this level of production is not possible, since the problem stated that the company could produce a maximum of 20 bikes per month.

Therefore, the correct answer is that the company cannot make \$150,000 profit in a month with its current production capacity.

PRACTICE PROBLEM FOR EXAMPLE 2

A local chiropractor has a small office where she cares for patients. She has \$8000 in fixed costs each month that cover her rent, basic salaries, equipment, and utilities. For each patient she sees, she has an average additional cost of about \$15. The chiropractor charges her patients or their insurance company \$80 for a visit.

a. Write an equation for the total monthly cost when v patient visits are done in a month.

b. What is the total monthly cost if the chiropractor has 100 patients visit during a month?

c. Write an equation for the monthly revenue when v patients visit a month.

d. Write an equation for the monthly profit the chiropractor makes if she has v patient visits in a month.

e. What is the monthly profit when 150 patients visit in a month?

f. How many patient visits does this chiropractor need to have in a month for her profit to be \$5000.00?

Example 2 shows that you should check each answer to determine whether or not it is a reasonable answer. Sometimes this requires some common sense; other times, a restriction that is stated in the problem should be considered.

In both of the previous examples, it is important to pay attention to the definition of each variable. The definitions of the variables helps to determine which variable value was given and which variable is to be solved for. Often, you will have to define the variables. Use meaningful variable names to make it easy to remember what they represent. For example,

- t = time in years
- h = hours after 12 noon
- p = population of San Diego (in thousands)
- P = profit of IBM (in millions of dollars)
- S = Salary (in dollars per hour)

Units, or how a quantity is measured, are very important in communicating what a variable represents. The meaning of $P = 100$ is very different if profit for IBM is measured in dollars and not millions of dollars. The meaning of $S = 6.5$ is different if S represents your salary for your first job out of college; it would be great if S were measured in millions of dollars per year and not dollars per hour. Units can make a large difference in the meaning of a quantity. When defining variables, always include units.

When solving an equation that represents something in an application, always check that the answer found is reasonable for the situation. Use the following Concept Investigation to practice determining which answers might be reasonable and which would not make sense in the situation given.

CONCEPT INVESTIGATION

Is that a reasonable solution?

In each part, choose the value that seems the most reasonable for the given situation. Explain why the other given value(s) do not make sense in that situation.

1. If P is the population of the United States in millions of people, which of the following is a reasonable value for P?

a. $P = -120$
b. $P = 300$
c. $P = 5{,}248{,}000{,}000$

2. If H is the height of an airplane's flight path in feet, which of the following is a reasonable value for H?

a. $H = -2000$
b. $H = 3{,}500{,}000$
c. $H = 25{,}000$

3. If P is the annual profit in dollars of a new flower shop the first year it opens, which of the following is a reasonable value for P?

a. $P = -40{,}000$
b. $P = 50{,}000$
c. $P = 3{,}000{,}000$

Steps to Solving Linear Equations

1. Simplify each side of the equation independently by performing any arithmetic and combining any like terms.
2. Move the variable terms to one side of the equation by using the addition or subtraction property of equality. Combine like terms.
3. Isolate the variable term by using the addition or subtraction property of equality.
4. Isolate the variable by using the multiplication or division property of equality.
5. Check the answer in the original problem.

From these steps, you can see that we use several properties to help us solve equations. We will refer to these properties as equation solving tools. These tools, along with many others we will work with throughout the book, make up the **Equation Solving Toolboxes** found at the back of the book. In Example 3, watch for how these tools are used throughout the solving process.

When solving equations that involve fractions, you can either work with the fractions throughout the solving process or eliminate the fractions during the first step. To clear the fractions during the first step, multiply both sides of the equation by the least common denominator and simplify. Then finish solving the equation as you would any other linear equation.

Linear Equations

Distributive Property

Use to simplify when an equation contains grouping symbols.

$$2(x + 4) = 3x - 5$$
$$2x + 8 = 3x - 5$$

Addition and Subtraction Property of Equality

Use to isolate a variable term.

$$\begin{aligned} 3x + 7 &= 20 \\ -7 \quad &\;\; -7 \\ \hline 3x &= 13 \end{aligned}$$

Multiplication and Division Property of Equality

Use to isolate a variable.

$$5x = 70$$
$$\frac{5x}{5} = \frac{70}{5}$$
$$x = 14$$

Example 3 Solving equations

Solve each equation. Check the answer.

a. $\frac{2}{3}x + \frac{5}{6} = 7$

b. $\frac{1}{4}(x + 5) = \frac{1}{2}x - 6$

c. $4t - 2(3.4t + 7) = 5t - 17.3$

SOLUTION

a. To eliminate the fractions, multiply both sides of the equation by the least common denominator 6 and then continue solving.

Step 1 Simplify each side of the equation independently by performing any arithmetic and combining like terms.

$$\frac{2}{3}x + \frac{5}{6} = 7$$ Simplify by clearing the fractions.

$$6\left(\frac{2}{3}x + \frac{5}{6}\right) = 6(7)$$ Multiply both sides by the least common denominator 6.

$$\frac{6}{1}\left(\frac{2}{3}x\right) + \frac{6}{1}\left(\frac{5}{6}\right) = 6(7)$$

$$\frac{{}^{2}\cancel{6}}{1}\left(\frac{2}{\cancel{3}}x\right) + \frac{\cancel{6}}{1}\left(\frac{5}{\cancel{6}}\right) = 6(7)$$ Reduce to eliminate the fractions.

$$4x + 5 = 42$$

Step 2 Move the variable terms to one side of the equation by using the addition or subtraction property of equality. Combine like terms.

This step is already complete since there are no variable terms on the right side of the equation.

Step 3 Isolate the variable term by using the addition or subtraction property of equality.

$$\begin{aligned} 4x + 5 &= 42 \\ -5 \quad &\;\; -5 \\ \hline 4x &= 37 \end{aligned}$$ Subtract 5 from both sides to isolate the variable term.

Step 4 Isolate the variable by using the multiplication and division property of equality.

$$\frac{4x}{4} = \frac{37}{4}$$ Divide both sides by 4 to isolate the variable.

$$x = \frac{37}{4}$$

Step 5 Check the answer in the original problem.

$$\frac{2}{3}\left(\frac{37}{4}\right) + \frac{5}{6} \stackrel{?}{=} 7$$ Substitute $\frac{37}{4}$ into x.

$$\frac{\cancel{2}}{3}\left(\frac{37}{\cancel{4}_{2}}\right) + \frac{5}{6} \stackrel{?}{=} 7$$

$$\frac{37}{6} + \frac{5}{6} \stackrel{?}{=} 7$$

$$\frac{42}{6} \stackrel{?}{=} 7$$

$$7 = 7$$ The answer checks.

Therefore, the answer is $x = \frac{37}{4} = 9.25$. This answer can be written as an improper fraction or a decimal as shown.

b. For this equation, distribute the $\frac{1}{4}$ first and then eliminate the fractions by multiplying both sides of the equation by the least common denominator 4.

Step 1

$$\frac{1}{4}(x + 5) = \frac{1}{2}x - 6$$

$$\frac{1}{4}x + \frac{1}{4}(5) = \frac{1}{2}x - 6$$ Distribute the $\frac{1}{4}$ to simplify the left side.

Skill Connection

Answer Format

In standard algebra problems, the convention is to answer with improper fractions if the original problem is given with improper fractions unless told otherwise. Likewise, if the original problem is given with decimals, answer with decimals unless told otherwise.